

FEM modeling of energy dissipation in SMRs

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Abstract

- Suspended Microchannel Resonator [1], [2], [3]
- Development of a 3D coupled fluid-structure interaction model to extract Quality Factor as function of fluid dynamic viscosity
- Comparison between numerical, theoretical [1] and experimental [2] results
- **Good match between experimental and numerical Q for first two modes**
- **Decreasing Q for increasing viscosity in contrast with theory**

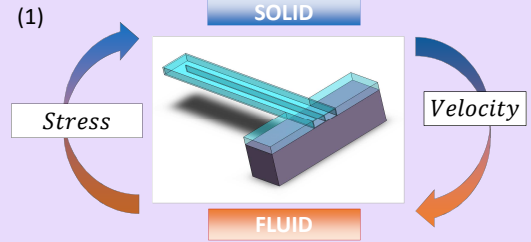


Fig. 1: Fluid-structure interaction is defined on the internal walls of the channel; a fixed constraint is imposed to the rigid channel ($x=0$). The cantilever is let free to vibrate ($x>0$). Linearized Navier-Stokes and Solid Mechanics equations are solved in COMSOL. The solid transfers momentum to the fluid, which sends back stresses to the cantilever, affecting its motion.

FEM model

- 3D eigenfrequency study in COMSOL Multiphysics®
- Device symmetry is exploited (fig.2a)
- Both 1-way-coupling and 2-way-coupling simulations are performed
- The quality factor is extracted as:

$$Q_{\text{Comsol}} = \frac{\text{Im}[\lambda]}{2\text{Re}[\lambda]}$$

where λ is the complex eigenvalue.

- The quality factor is scaled according to the analytical model proposed by Sader in [1], in function of the Reynolds number β :

$$Q = F(\beta) \frac{\rho_c}{\rho_f} \left(\frac{h_c}{h_f} \right) \left(\frac{b_c}{b_f} \right) \left(\frac{L}{h_f} \right)^2, \quad \beta = \frac{\rho_f \omega h_f^2}{\mu}$$

- Parameters studied: compressibility ($\gamma = \left(\frac{\omega L}{c} \right)^2$ is the normalized acoustic wavenumber), dynamic viscosity, off-axis placement Z_0 , Poisson ratio, mode number.

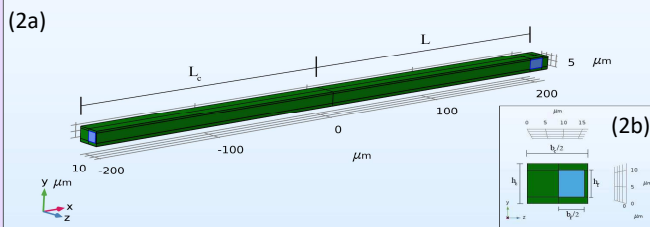


Fig.2a: COMSOL Model (half geometry) of Device A [1]: $h_f=8 \mu\text{m}$, $h_c=12 \mu\text{m}$, $b_f=16 \mu\text{m}$, $b_c=33 \mu\text{m}$, $L=204 \mu\text{m}$, $L_c=210 \mu\text{m}$, cantilever length=210 μm , $Z_0=0.06$. In green the elastic domain, in blue the fluid domain.

Z_0 is the off-axis placement of the fluidic channel with respect to the beam neutral axis.

Fig.2b: Cross-section of half geometry of Device A [1] (symmetry boundary condition is exploited)

Theoretical model [1]

- 2D theoretical model is only due to fluid motion and viscous forces, through the rate-of-strain tensor \mathbf{e} , defined as: $E_{ij} = \frac{1}{2}(\partial_j v_i + \partial_i v_j)$

- Quality factor is computed as:

$$Q = 2\pi \frac{E_{\text{stored}}}{E_{\text{diss/cycle}}} \bigg|_{\omega_R}$$

- Strong effect of:

- compressibility [1]
- channel off-placement Z_0 [1]
- Poisson's ratio [3]
- mode number [2]

$$(3) \quad v = -i\omega \left(W(x|\omega)z - \left[z_0 + \frac{h_{\text{fluid}}}{2} \right] \frac{\partial W}{\partial x} \right)$$

$$z = z_0 + \frac{h_{\text{fluid}}}{2}$$

$$z = z_0 - \frac{h_{\text{fluid}}}{2}$$

$$v = -i\omega \left(W(x|\omega)z - \left[z_0 - \frac{h_{\text{fluid}}}{2} \right] \frac{\partial W}{\partial x} \right)$$

Fig. 3: 2D theoretical model; Euler-Bernoulli beam equations imposed as boundary conditions on the top and bottom wall; x is the coordinate along the length of the beam, z_0 is the off-placement of the channel with respect to the beam neutral axis [1].

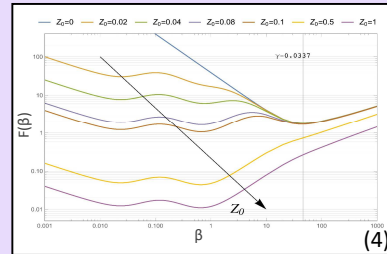


Fig. 4: Theoretical Normalized Quality Factor $F(\beta)$ for various normalized off-placements Z_0 of the channel in the compressible case ($\gamma=0.0337$) for $L=L_c$; the theoretical model predicts an increasing $F(\beta)$ for increasing viscosity (decreasing β) and lower $F(\beta)$ for higher off-placements of the channel with respect to the beam neutral axis. For $Z_0 < 0.2$ this effect is stronger for $\beta < 10$.

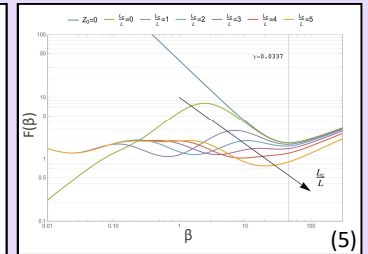


Fig. 5: Theoretical Normalized Quality Factor $F(\beta)$ for various rigid lead channel lengths L_c in the compressible case ($\gamma=0.0337$) and $Z_0=0.1$; the theoretical model predicts a surprisingly different behavior when $L_c=0$. The local maxima and minima of $F(\beta)$ are strongly affected by L_c .

Results

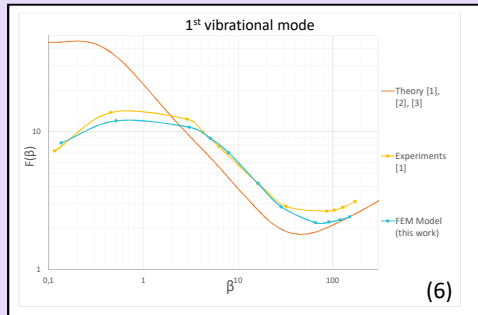


Fig. 6: Comparison of Normalized Quality Factor $F(\beta)$ as a function of Reynolds Number between theoretical [1], [2], [3], experimental [1], [2] and numerical results for Device A ($h_f=8 \mu\text{m}$, $h_c=12 \mu\text{m}$, $b_f=16 \mu\text{m}$, $b_c=33 \mu\text{m}$, $L=204 \mu\text{m}$, $L_c=210 \mu\text{m}$, cantilever length=210 μm , $Z_0=0.06$, normalized wavenumber $\gamma=0.12$, Poisson's ratio=0.25) for Mode 1. Viscosity spans from 1 mPa·s to 1000 mPa·s and is inversely proportional to Reynolds Number.

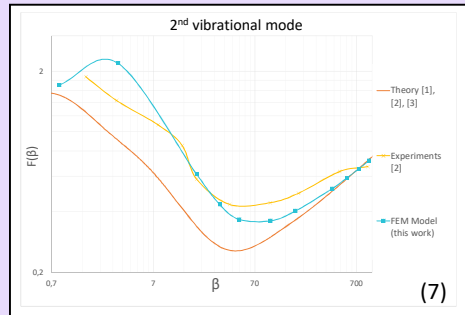


Fig. 7: Comparison of Normalized Quality Factor $F(\beta)$ as a function of Reynolds Number between theoretical [1], [2], [3], experimental [1], [2] and numerical results for Device A ($h_f=8 \mu\text{m}$, $h_c=12 \mu\text{m}$, $b_f=16 \mu\text{m}$, $b_c=33 \mu\text{m}$, $L=204 \mu\text{m}$, $L_c=210 \mu\text{m}$, cantilever length=210 μm , $Z_0=0.06$, normalized wavenumber=0.12, Poisson's ratio=0.25) for Mode 2. Viscosity spans from 1 mPa·s to 1000 mPa·s and is inversely proportional to Reynolds Number.

Conclusions

- Good agreement between experimental and numerical results for $\beta \in (1, 1000)$ for first two modes
- Contrasting behavior at high viscosities between theoretical and numerical results ($\beta < 1$)
- Dependence of Q on Z_0 , L_c , compressibility, Poisson's ratio and mode number for $\beta < 1$.
- Need of improvement of 2-way-coupling modelling

References

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